RC FILTERS

Electronic Measurements Lab

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Preliminaries

In this lab you will learn how to measure the transfer function of a filter. Have a look at your workbench, you will need the following laboratory equipment (check here for specifications), components and cables:

- Filters board
- Oscilloscope
- Function generator
- 1 BNC male to BNC male coaxial cable
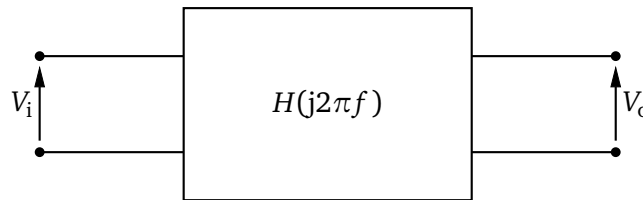- 2 crocodile clips to BNC male cable (or 2 oscilloscope probes)

1   Introduction

1.1   Bode plots

A generic linear filter (figure 1) is characterised by its (complex) transfer function

$$H(\mathrm{j}2\pi f) = \frac{V_\mathrm{o}}{V_\mathrm{i}},$$

where V_i is the phasor associated with a sinusoidal input voltage at frequency f, and V_o is the phasor associated with the corresponding sinusoidal output voltage.

Figure 1

$H(\mathrm{j}2\pi f)$ is thus a complex number which changes with f. The Bode plots are a graphical representation of the quantities $G(f) = 20\log|H(\mathrm{j}2\pi f)|$ and $\varphi(f) = \arg H(\mathrm{j}2\pi f)$ as a function of frequency (fig. 2). $G(f)$ is the magnitude of the transfer function, expressed in decibel (dB), and $\varphi(f)$ is the phase. The frequency axis is logarithmic.
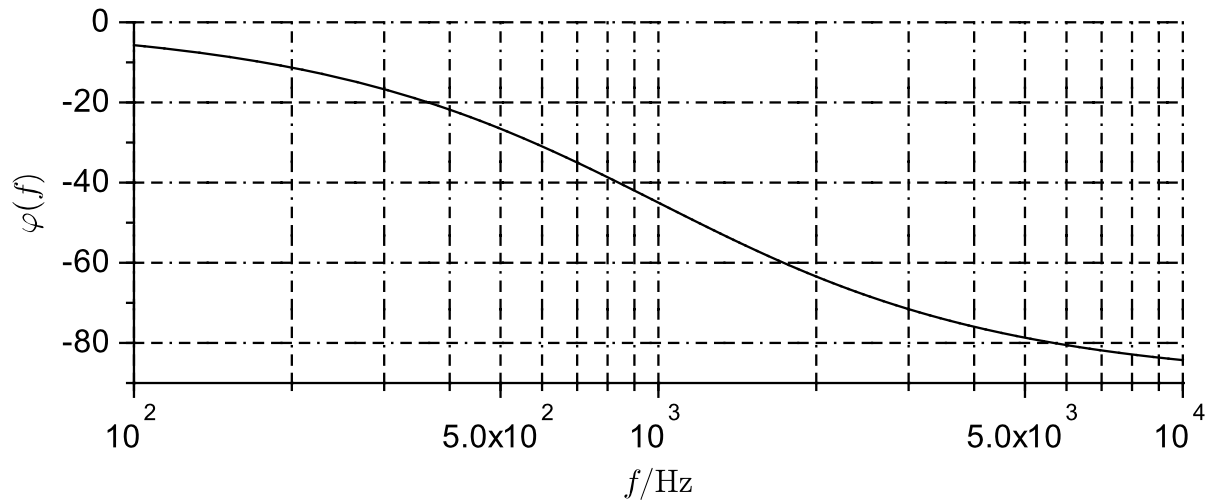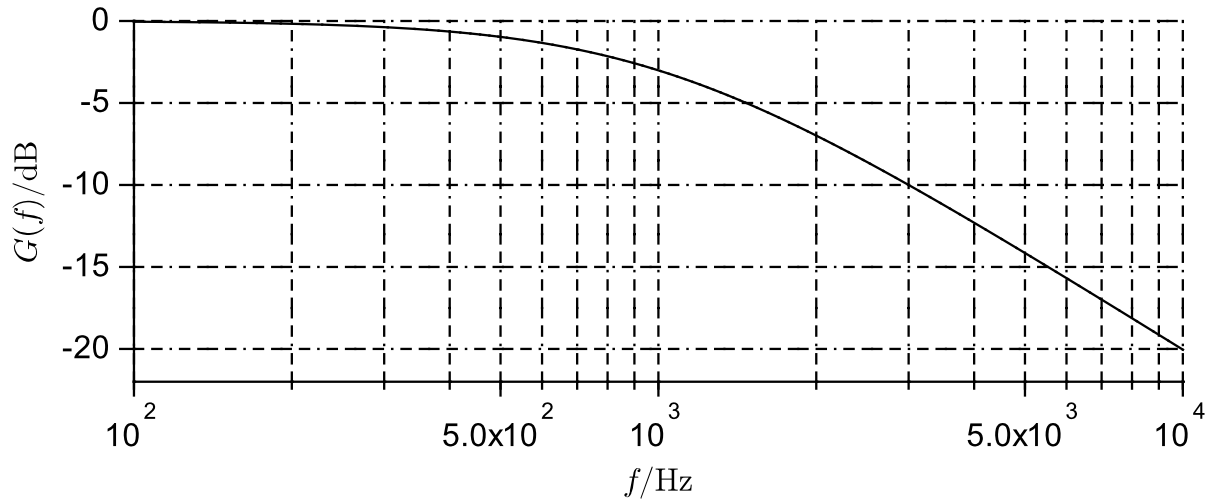
Figure 2: Example Bode plots.

In general, to measure $G(f)$ and $\varphi(f)$, one should:

1. Choose a set of frequencies $f_1, f_2, \ldots, f_N$ which covers the frequency range where the filter is to be characterised.
2. Inject a sinusoidal signal at frequency f_1 into the filter input.
3. Measure, for example with an oscilloscope, the ratio of the output voltage amplitude (e.g. peak-to-peak amplitude) to the input voltage amplitude. This ratio yields $G(f_1)$.
4. Measure the phase shift of the output voltage with respect to the input voltage. The value of this phase shift yields $\varphi(f_1)$.
5. Repeat steps 2–4 for each frequency $f_2, \ldots, f_N$.

Before going on with the measurement, however, read carefully the next two paragraphs which describe the filters and the function generator.

1.2 Filters board

The filters board contains several filters: in particular, you will characterise the low- and the high-pass filters (fig. 3). These filters are first-order *RC* filters and their circuit diagrams and ideal transfer functions are given in figures 4 and 5.
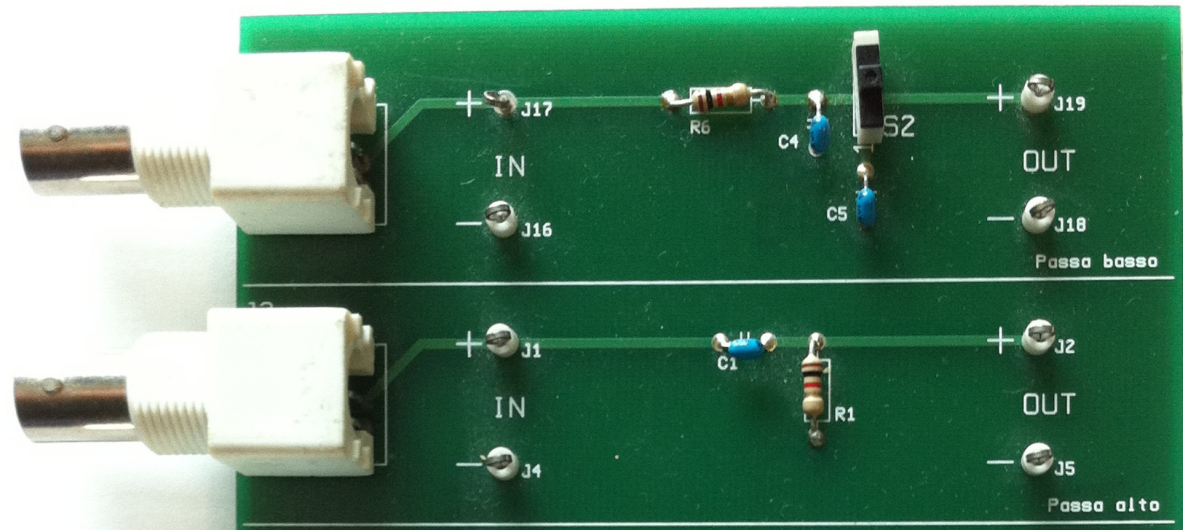
Figure 3: A detail of the filters board showing, from top to bottom, the low- and high-pass filters. The terminals labelled with "−" are the ground terminals.
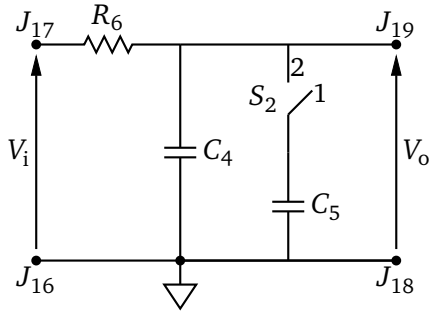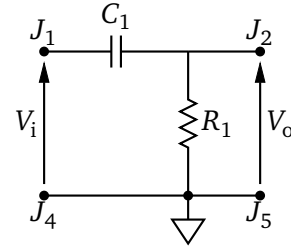
Figure 4: Circuit diagram of the low-pass RC filter of figure 3. If the switch S_2 is in position 1, only C_4 is connected; otherwise, in position 2, C_4 and C_5 are connected in parallel.

The ideal transfer function is

$$H_{\mathrm{LP}}(\mathrm{j}2\pi f) = \frac{1}{1 + \mathrm{j}2\pi f\, \tau_{\mathrm{LP}}}$$

with $\tau_{\mathrm{LP}} = R_6 C_4$ or $\tau_{\mathrm{LP}} = R_6(C_4 + C_5)$.

Figure 5: Circuit diagram of the high-pass CR filter of figure 3.

The ideal transfer function is

$$H_{\mathrm{HP}}(\mathrm{j}2\pi f) = \frac{\mathrm{j}2\pi f\, \tau_{\mathrm{HP}}}{1 + \mathrm{j}2\pi f\, \tau_{\mathrm{HP}}}$$

with $\tau_{\mathrm{HP}} = R_1 C_1$.

For this kind of filters, the $-3\,\text{dB}$ cut-off frequency is given by

$$f_\text{T} = \frac{1}{2\pi\tau} \,,$$

where $\tau = \tau_\text{LP}$ or $\tau = \tau_\text{HP}$, depending on the filter, is the time constant. At the cut-off frequency, $G(f)$ falls by $3\,\text{dB}$ below its maximum value (which is $0\,\text{dB}$), i.e $G(f_\text{T}) = -3\,\text{dB}$. Alternatively, at this frequency,

$$|V_\text{o}| = \frac{1}{\sqrt{2}}|V_\text{i}| \,.$$

1.3   Function generator

A function generator is an instrument which can generate several voltage waveforms with variable frequency and amplitude. Typically, function generators can generate sine, square, triangle and pulse waves, possibly with amplitude and frequency modulations, but many modern generators can also generate arbitrary waveforms programmed by the user. In addition, function generators can add to the output waveform a DC component, which is called *offset*.

The output of the function generator can be modelled as an ideal voltage source in series with an internal resistance (fig. 6). The amplitude that can be read from the front panel display is typically the peak-to-peak amplitude of the output voltage V_o when the output is loaded with a resistance $R_{load} = R_g$ (fig. 7). If the output is unloaded, the actual peak-to-peak amplitude of the output voltage is approximately twice that indicated by the generator.
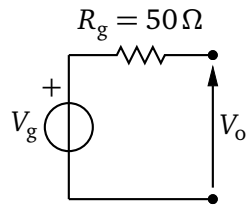
Figure 6: Circuit model of a function generator. V_g is the open circuit output voltage; R_g is the internal resistance of the generator and has a typical value of $50\,\Omega$.
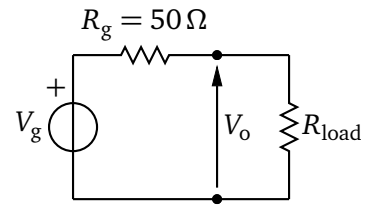
Figure 7: Loaded function generator. If $R_{load} = R_g$, the output voltage V_o is half that of the open circuit output voltage V_g.

1. By means of a BNC to BNC cable, connect the function generator to the oscilloscope and observe what happens when you change the waveform parameters (waveform type, amplitude and frequency) from the function generator front panel.
2. Add an offset to the output waveform and see what happens when you change its value. Make sure that the oscilloscope is DC coupled.

2 Low-pass RC filter

2.1 Nominal time constant and cut-off frequency

1 Find the nominal value of the resistance R_6. The nominal resistance value can be read from the color bands marked on the resistor body, by means of the color code given in figure 8.

2 Find the nominal values of the capacitances C_4 and C_5. The capacitance value can be read from the three-digit code marked on the capacitor body: the first two digits give the value (in pF), while the last digit gives the exponent of a base 10 multiplier. For instance, 224 means $22 \times 10^4 \, \text{pF} = 220 \, \text{nF}$.

3 From the above determined values, calculate the nominal time constants and cut-off frequencies for both the positions of S_2.

4 Knowing that the resistance has a 5 % tolerance and that the capacitances have a 10 % tolerance, determine the uncertainty of the parameters calculated at the previous point.
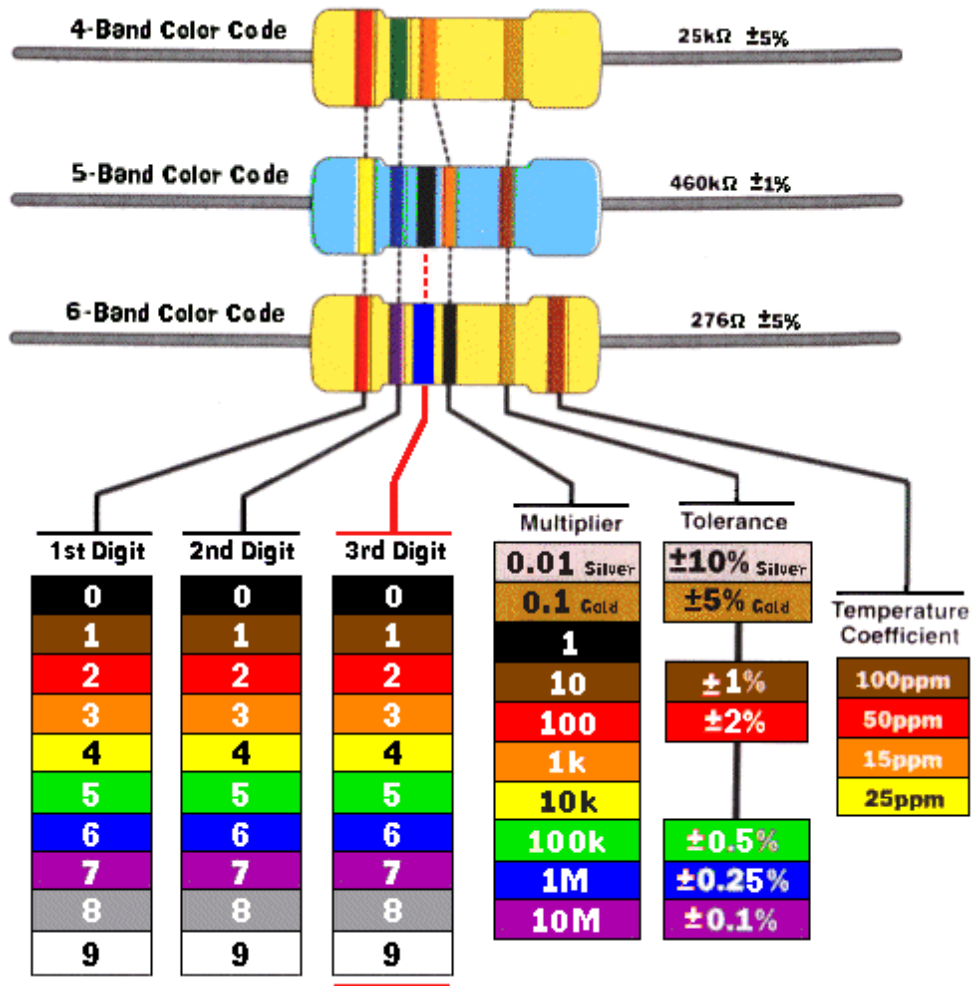
Figure 8: Resistor color code (picture source).

2.2 Transfer function

1 Set the switch S_2 on the filters board to position 1.

2 Set the function generator for a sinusoidal output voltage with an amplitude of a few volts and no offset.

3 Set the oscilloscope to display both channels 1 and 2. Both inputs should be DC-coupled (should the function generator have a significant offset, use AC-coupled inputs). Centre the traces vertically on the screen.

4 By means of the BNC-to-BNC cable, connect the function generator to the input of the low-pass filter.

5 Connect the two channels of the oscilloscope respectively to the input and the output of the filter (use the crocodile clips to BNC cables). **Important:** The black clips should be connected to the ground test points J_{16} and J_{18}, otherwise you will short-circuit the output of the function generator.

6 Measure $G(f)$ and $\varphi(f)$ over the frequency range from 500 Hz to 500 kHz (see also figure 9 for a definition of the parameters). We suggest to make measurements at the following frequency points, but you can also make other choices: 500 Hz, 800 Hz, 1200 Hz, 2000 Hz, 3000 Hz, 5 kHz, 8 kHz, 12 kHz, 20 kHz, 30 kHz, 50 kHz, 80 kHz, 120 kHz, 200 kHz, 300 kHz and 500 kHz. If you make other choices, try to space the points so that they roughly form a geometric progression: in this way, the points will be equally spaced on a logarithmic scale.

7 Search the -3 dB cut-off frequency by continuously varying the input frequency; compare this value with the one obtained in §2.1.

8 Repeat the above measurements with S_2 in position 2.
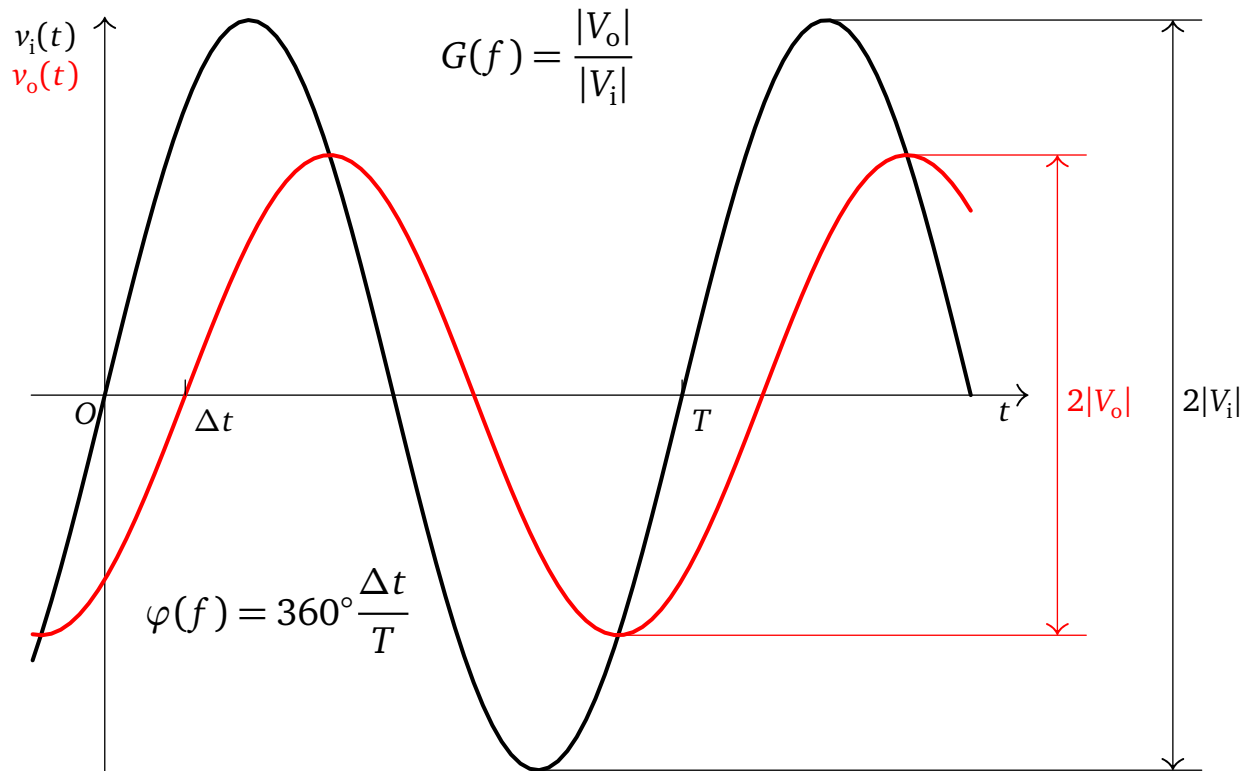
Figure 9: This figure shows an example of input (black) and output (red) voltage waveforms. Δt is the time interval (with sign) between two nearby zero crossings. Here Δt should be considered negative because the output signal *lags* (i.e. it is delayed) with respect to the input one.

2.3 Square wave response

1 Set the function generator for a square-wave output. Choose an output frequency with period $T \gg \tau_{LP}$.

2 Measure the rise-time t_r of the output waveform. Compare the measured value with the theoretical value $t_r \approx 2.2\tau_{LP}$.

3   High-pass CR filter

Repeat the steps of §2.1 and §2.2 for the high pass filter.